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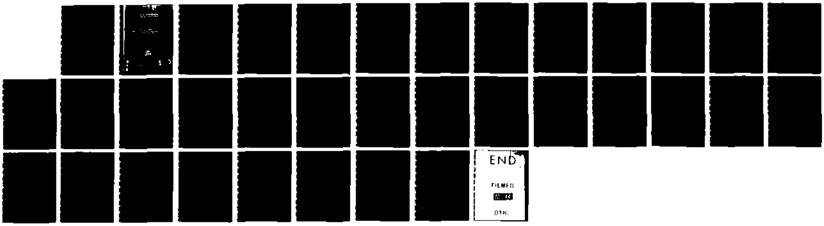

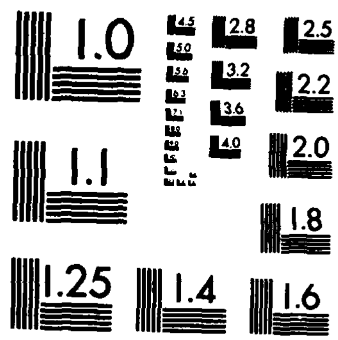

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ESTIMATION OF TWO-PARAMETER LOGISTIC ITEM RESPONSE CURVES

Robert K. Tsutakawa

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December 1983

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper presents a method for estimating certain characteristics of test items which are designed to measure ability, or knowledge, in a particular area. Under the assumption that ability parameters are sampled from a normal distribution, the EM algorithm is used to derive maximum likelihood estimates of item parameters of the two-parameter logistic item response curves. The observed information matrix is used to approximate the covariances of these estimates.

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Responses to a questionnaire on general arthritis knowledge are used to illustrate the procedure and simulated data are used to compare the actual versus estimated items parameters. A computational note is included to facilitate the extensive numerical work required to implement the procedure.

ESTIMATION OF TWO-PARAMETER

LOGISTIC ITEM RESPONSE CURVES

by

Robert K. Tsutakawa

University of Missouri-Columbia

ACKNOWLEDGEMENTS

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INTRODUCTION

We consider dichotomous responses by subjects to a set of test items, when the responses are assumed to follow the two parameter logistic model. Under the assumption that ability parameters are randomly sampled from a normal distribution with unknown parameters, empirical Bayes types estimates of ability and item parameters were proposed and illustrated for the one-parameter logistic (Rasch) model by Rigdon and Tsutakawa (1983). Here we will consider one of these estimators, namely MLF, where the extension to multiparameter item response models is straight-forward and asymptotic methods are available from the theory of maximum likelihood estimation.

A standard procedure for estimating both ability and item parameters in the absence of prior distributions is maximum likelihood (cf. Lord 1980). Maximum likelihood estimation when the ability parameters have a common prior distribution have been studied by a number of authors, including, among others, Bock and Lieberman (1970), Andersen and Madsen (1977), Sanathanan and Blumenthal (1978), and Bock and Aitkin (1981). The latter two papers use different forms of the EM algorithm (Dempster, Laird, and Rubin 1977) to greatly simplify what previously appeared to be an insurmountable numerical problem.

This paper shows that by using the extended form of the EM algorithm, which does not require the presence of sufficient statistics, the computational difficulty is reduced to solving a series of 2 equations with 2 unknown, corresponding to the 2 parameters of the logistic curve. Although the equations must be numerically solved by some iterative method, the task is a vast simplification over methods where all parameters are found simultaneously. Under the

assumption that the ability parameters are independent and identically distributed, the classical maximum likelihood theory applies and the inverse of the observed information matrix may be used as an approximate covariance matrix of the estimated item parameters. The successful use of our method requires a fair amount of computation. Most of the computational burden is in accurately evaluating a large number of single intergals, which are basically expectations with respect to posterior distributions of abilities.

After describing our model and assumptions, we discuss the implementation of the EM algorithm to our problem. The equations at each iteration of the algorithm is then related to the likelihood equations. Responses to a questionnarie on arthritis knowledge will then be used to illustrate our method and simulations will be used to illustrate the reliability of the observed information matrix for assessing the errors in the item parameter estimates. Expressions of derivatives needed to carry out the computation are summarized in the last section.

ITEM RESPONSE MODEL

Consider n subjects responding to k test items designed to measure the ability of the subjects in a particular area. Let $\underset{\sim}{Y}$ be an $n \times k$ matrix of binary responses where $Y_{ij} = 1$ or 0 accordingly as the ith subject's response to item j is correct or incorrect, $i = 1,\cdots,n$ and $j = 1,\cdots,k$. A general framework in which to study such item responses is to consider some probability model,

$$P_{ij} = P(Y_{ij} = 1|\theta_i,\underset{\sim}{\xi}_j), \tag{2.1}$$

depending on a real valued ability parameter of the ith subject, θ_i, and a real or vector valued item parameter of the jth item, $\underset{\sim}{\xi}_j$. Given $\underset{\sim}{\theta}^T = (\theta_1, \cdots, \theta_n)$ and $\underset{\sim}{\xi}^T = (\underset{\sim}{\xi}_1^T, \cdots, \underset{\sim}{\xi}_k^T)$, where T denotes transpose, conditional independence among the responses in $\underset{\sim}{Y}$ is assumed, so that the joint probability of $\underset{\sim}{Y} = \underset{\sim}{y}$, given $(\underset{\sim}{\theta}, \underset{\sim}{\xi})$ is

$$P(\underset{\sim}{y}|\underset{\sim}{\theta},\underset{\sim}{\xi}) = \prod_{i=1}^{n} \prod_{j=1}^{k} p_{ij}^{y_{ij}} (1 - p_{ij})^{1-y_{ij}}. \tag{2.2}$$

We further assume that $\underset{\sim}{\theta}$ may be treated as a random sample from some prior distribution with pdf $p(\theta|\underset{\sim}{\lambda})$ having parameter $\underset{\sim}{\lambda}$.

In this paper we shall be discussing the estimation of $\underset{\sim}{\xi}$ in the special case of the two-parameter logistic model, defined by

$$P(y_{ij}|\theta_i,\alpha_j,\beta_j) = \frac{e^{y_{ij}\alpha_j(\theta_i - \beta_j)}}{1 + e^{\alpha_j(\theta_i - \beta_j)}}, \tag{2.3}$$

$y_{ij} = 0,1$, where $\underset{\sim}{\xi}_j$ is now the vector (α_j,β_j) with $0 < \alpha_j < \infty$ and $-\infty < \beta_j < \infty$. For the prior of θ, we consider the normal distribution with mean μ and variance σ^2 so that $\underset{\sim}{\lambda} = (\mu,\sigma^2)$. However, due to the nonuniqueness of this parametrization we shall add the constraints $\mu=0$ and $\sigma^2=1$. We will thus assume that the prior pdf of θ is

$$p(\theta) = (2\pi)^{-1/2} \exp(-\theta^2/2), \tag{2.4}$$

$-\infty < \theta < \infty$. It can be shown (Tsutakawa 1982), that there is an equivalence between this parametrization and the more familiar one

where the constraints are

$$\sum_{j=1}^{k} \beta_j = 0 \quad \text{and} \quad \prod_{j=1}^{k} \alpha_j = 1,$$

with no restriction on (μ,σ^2).

For the above model, the joint distribution of $(\underset{\sim}{Y}_i^T,\theta_i)$, where $\underset{\sim}{Y}_i^T = (Y_{i1},\ldots,Y_{ik})$, is given by

$$f(\underset{\sim}{y}_i^T,\theta_i|\underset{\sim}{\xi}) = p(\theta_i) \prod_{j=1}^{k} P(y_{ij}|\theta_i,\alpha_j,\beta_j) \tag{2.5}$$

and the joint distribution of $(\underset{\sim}{Y},\underset{\sim}{\theta})$ by

$$f(\underset{\sim}{y},\underset{\sim}{\theta}|\underset{\sim}{\xi}) = \prod_{j=1}^{n} f(y_i^T,\theta_i|\underset{\sim}{\xi}). \tag{2.6}$$

Thus the marginal probability of response $\underset{\sim}{y}_i$ for the ith subject is

$$P(\underset{\sim}{y}_i^T|\underset{\sim}{\xi}) = \int f(y_i^T,\theta_i|\underset{\sim}{\xi})d\theta_i \tag{2.7}$$

and for all subjects

$$P(\underset{\sim}{y}|\underset{\sim}{\xi}) = \prod_{i=1}^{n} P(\underset{\sim}{y}_i^T|\underset{\sim}{\xi}). \tag{2.8}$$

The loglikelihood function of $\underset{\sim}{\xi}^T = \{(\alpha_1,\beta_1),\ldots,(\alpha_k,\beta_k)\}$ is thus given by

$$L(\underset{\sim}{\xi}) = \sum_{i=1}^{n} \log \int p(\theta_i) \prod_{j=1}^{k} P(y_{ij}|\theta_i,\alpha_j,\beta_j)d\theta_i. \tag{2.9}$$

PARAMETER ESTIMATION VIA THE EM ALGORITHM

In this section we outline the use of EM algorithm for obtaining $\hat{\underset{\sim}{\xi}}$, the maximum likelihood estimate of $\underset{\sim}{\xi}$, when the loglikelihood function is given by (2.9). Basically the algorithm consists of iteratively deriving the value of $\underset{\sim}{\xi}$ which maximizes the posterior expectation

$$T(\underset{\sim}{\xi}) = E\{\log f(\underset{\sim}{y},\underset{\sim}{\theta}|\underset{\sim}{\xi})|\ \underset{\sim}{y},\underset{\sim}{\xi}^{\circ}\}, \tag{3.1}$$

given $\underset{\sim}{y}$ and a provisional estimate $\underset{\sim}{\xi}^{\circ}$, where $\underset{\sim}{\xi}^{\circ}$ is initially some approximation to $\hat{\underset{\sim}{\xi}}$ and is replaced at the end of each iteration by the maximizing value of $\underset{\sim}{\xi}$.

For the two-parameter logistic model (2.3), each iteration reduces to solving the equations,

$$\sum_{i=1}^{n} y_{ij} = \sum_{i=1}^{n} \int \frac{p(\underset{\sim}{\theta}_i|\underset{\sim}{y}_i,\underset{\sim}{\xi}^{\circ})}{1+\exp\{-\alpha_j(\theta_i-\beta_i)\}}\, d\theta_i\ , \tag{3.2}$$

$$\sum_{i=1}^{n} y_{ij} \int \theta_i p(\theta_i|\underset{\sim}{y}_i,\underset{\sim}{\xi}^{\circ})\, d\theta_i = \sum_{i=1}^{n} \int \frac{\theta_i p(\theta_i|\underset{\sim}{y}_i,\underset{\sim}{\xi}^{\circ})}{1+\exp\{-\alpha_j(\theta_i-\beta_i)\}}\, d\theta_i,$$

for (α_j,β_j), separately for $j = 1,\ldots,k$, where

$$\underset{\sim}{\xi}^{\circ} = \{(\alpha_1^{\circ},\beta_1^{\circ}),\ldots,(\alpha_k^{\circ},\beta_k^{\circ})\}^T, \quad \text{and}$$

$$p(\theta_i|\underset{\sim}{y}_i,\underset{\sim}{\xi}^{\circ}) = \frac{f(\underset{\sim}{y}_i,\theta_i|\underset{\sim}{\xi}^{\circ})}{P(\underset{\sim}{y}_i|\underset{\sim}{\xi}^{\circ})}\ , \tag{3.3}$$

the posterior pdf of θ_i, given $\underset{\sim}{\xi} = \underset{\sim}{\xi}^{\circ}$ and observation $\underset{\sim}{y}_i$, $i=1,\ldots,n$. The solution at the end of each iteration is used as the value of $\underset{\sim}{\xi}^{\circ}$ in the next iteration, and the process continues till convergence is attained.

Since the solution to the equation (3.2) has no simple expression it must be derived numerically by iterative methods such as the one by Marquardt (1963). Moreover, the integrals must be evaluated by numerical techniques such as Gauss-Hermite. The expressions for the derivative needed for the implementation these methods are summarized in the last section.

It is instructive to compare equations (3.2) with the likelihood equation which $\hat{\underset{\sim}{\xi}}$ satisfies. The likelihood equations, which are obtained by setting the first partial derivatives of (2.9) with respect to α_j and β_j equal to zero, are

$$\sum_{i=1}^{n} y_{ij} = \sum_{i=1}^{n} \int \frac{p(\theta_i | \underset{\sim}{y}_i, \underset{\sim}{\xi})}{1 + \exp\{-\alpha_j(\theta_i - \beta_j)\}} \, d\theta_i \, ,$$

$$\sum_{i=1}^{n} y_{ij} \int \theta_i p(\theta_i | y_i, \underset{\sim}{\xi}) \, d\theta_i = \sum_{i=1}^{n} \int \frac{\theta_i p(\theta_i | \underset{\sim}{y}_i, \underset{\sim}{\xi})}{1 + \exp\{-\alpha_j(\theta_i - \beta_j)\}} \, d\theta_i, \tag{3.4}$$

$j=1,\ldots,k$, where $\underset{\sim}{\xi} = \{(\alpha_1,\beta_1),\ldots,(\alpha_k,\beta_k)\}$ The only difference between the two systems of equations (3.2) and (3.4) is in the role $\underset{\sim}{\xi}$ and $\underset{\sim}{\xi}^{\circ}$. In (3.2) $\underset{\sim}{\xi}^{\circ}$ is a fixed known value at each iteration, whereas in (3.4) $\underset{\sim}{\xi}$ is an unknown to be solved. Solving (3.4) directly requires simultaneously finding 2k unknowns in 2k equations.

On the other hand, each iteration of the EM algorithm only requires finding a series of 2 unknowns in 2 equations. Note that once convergence is attained using the EM algorithm, the successive values of $\underset{\sim}{\xi}^{\circ}$ remain unchanged and satisfy the likelihood equations (3.4).

ERRORS IN PARAMETER ESTIMATES

Since the vector valued observations $\underset{\sim}{Y}_1, \underset{\sim}{Y}_2, \ldots, \underset{\sim}{Y}$ are independent with a common distribution (2.7), we appe to the asymptotic properties of the maximum likelihood e ator to assess the reliability of $\hat{\underset{\sim}{\xi}}$. For large samples, $\hat{\underset{\sim}{\xi}}$ is approximately normal with mean $\underset{\sim}{\xi}$ and covariance $I^{-1}(\underset{\sim}{\xi})$, the reciprocal of the Fisher information matrix $I(\underset{\sim}{\xi})$. In practice $\underset{\sim}{\xi}$ is unknown and $I(\underset{\sim}{\xi})$ is difficult to compute. We therefore propose estimating $I(\underset{\sim}{\xi})$ by the observed information matrix $\mathcal{I}(\hat{\underset{\sim}{\xi}})$, defined as the negative second derivative matrix of the loglikelihood function (2.9). Justifications for this approximation are given in Efron and Hinkley (1978). Expressions of $\mathcal{I}(\underset{\sim}{\xi})$ suitable for numerical computation are summarized below under computational notes. Numerical examples of this approximation are shown in the next two sections.

APPLICATION TO ARTHRITIS KNOWLEDGE

We illustrate our method using responses from hospital patients to a 50 item questionnaire on general information regarding arthritis. The questionnarie was completed by n = 167 subjects. Of the original 50 items, 3 were deleted since their parameters could not be estimated, and we thus used the remaining k = 47 items.

Estimates of (α_j, β_j) and there covariances are summarized in Table 1.

Insert Table 1 about here

SIMULATION

To obtain some indication of performance, our method was applied to a randomly generated data set, $\underset{\sim}{Y}$, using n = 200 ability parameters randomly selected from a standard normal distribution and k = 50 pairs of item parameters (α_j, β_j) whose values are in the range typically encountered in pratice. Figures 1, 2 and 3 present plots of the estimated versus simulated values of α, β, and θ. The estimates for θ are posterior means obtained from (3.3) with $\underset{\sim}{\xi}^\circ = \hat{\underset{\sim}{\xi}}$, as in Rigdon and Tsutakawa (1983). We note a fairly high correlation of .988 and .974 for β and θ, respectively, but a somewhat lower one, .881, for α. The estimated covariances for $(\hat{\alpha}, \hat{\beta})$ are summarized in Table 2 together with standardized errors $(\hat{\alpha} - \alpha)/SD(\hat{\alpha})$ and $(\hat{\beta} - \beta)/SD(\hat{\beta})$, where SD's are square roots of the variances approximated by $I^{-1}(\hat{\underset{\sim}{\xi}})$. The simulation was repeated using n = 400 and k = 25, with similar results on the standardized errors. Although these results appear quite plausible, until we obtain more experience using $I(\hat{\underset{\sim}{\xi}})$ the method should be used with caution for data sets of smaller sizes or when the model may not be appropriate.

Insert Figure 1,2, & 3 and Table 2 about here

COMPUTATIONAL NOTES

We now summarize expressions for the first two derivatives of the loglikelihood function (2.9) and the posterior expectation (3.1), which may be used to evaluate $I(\hat{\underset{\sim}{\xi}})$ and to maximize $T(\underset{\sim}{\xi})$, respectively.

For each $i = 1,\ldots,n$ and $u = 1,\ldots,k$ let

$$g_1(i,u,\theta) = \frac{\partial P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial \alpha_u} \Big/ P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$g_2(i,u,\theta) = \frac{\partial P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial \beta_u} \Big/ P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$h_{11}(i,u,\theta) = \frac{\partial^2 P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial \alpha_u^2} \Big/ P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$h_{12}(i,u,\theta) = \frac{\partial^2 P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial \alpha_u \, \partial \beta_u} \Big/ P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$h_{22}(i,u,\theta) = \frac{\partial^2 P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial \beta_u^2} \Big/ P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$\phi_{\theta u} = \{1 + \exp[-\alpha_u(\theta - \beta_u)]\}^{-1}, \quad \text{and} \quad \psi_{\theta u} = 1 - \phi_{\theta u}.$$

By taking derivatives it is readily shown that

$$g_1(i,u,\theta) = (y_{iu} - \phi_{\theta u})(\theta - \beta_u),$$

$$g_2(i,u,\theta) = -\alpha_u(y_{iu} - \phi_{\theta u}),$$

$$h_{11}(i,u,\theta) = (y_{iu} - \phi_{\theta u})(\psi_{\theta u} - \phi_{\theta u})(\theta - \beta_u)^2,$$

$$h_{12}(i,u,\theta) = (y_{iu} - \phi_{\theta u})[1 + \alpha_u(\psi_{\theta u} - \phi_{\theta u})(\theta - \beta_u)],$$

and

$$h_{22}(i,u,\theta) = (y_{iu} - \phi_{\theta u})\,\alpha_u^2(\psi_{\theta u} - \phi_{\theta u}).$$

Now define the following posterior expectations of the derivatives and their products by

$$\bar{g}_s(i,u) = \int g_s(i,u,\theta)p(\theta|\underset{\sim}{y}_i,\underset{\sim}{\xi})d\theta, \tag{7.1}$$

$$\bar{h}_{st}(i,u) = \int h_{st}(i,u,\theta)p(\theta|\underset{\sim}{y}_i,\underset{\sim}{\xi})d\theta, \tag{7.2}$$

$$\bar{d}_{st}(i,u,v) = \int g_s(i,u,\theta)g_t(i,v,\theta)p(\theta|\underset{\sim}{y}_i,\underset{\sim}{\xi})d\theta, \tag{7.3}$$

$s,t = 1,2,\quad u,v = 1,\ldots,k,\quad u \neq v,$ where $p(\theta|\underset{\sim}{y}_i,\underset{\sim}{\xi})$ is defined by (3.3).

Then the first and second derivatives of the loglikelihood function $L(\underset{\sim}{\xi})$, defined by (2.9), may be expressed by

$$\frac{\partial L(\underset{\sim}{\xi})}{\partial \alpha_u} = \sum_{i=1}^{n} \bar{g}_1(i,u), \tag{7.4}$$

$$\frac{\partial^2 L(\underset{\sim}{\xi})}{\partial \beta_u} = \sum_{i=1}^{n} \bar{g}_2(i,u), \tag{7.5}$$

$$\frac{\partial^2 L(\underset{\sim}{\xi})}{\partial \alpha_u \partial \alpha_v} = \begin{cases} \sum_{i=1}^{n} \{\bar{h}_{11}(i,u) - \bar{g}_1^2(i,u)\} & \text{if } u = v \\ \sum_{i=1}^{n} \{\bar{d}_{11}(i,u,v) - \bar{g}_1(i,u)\bar{g}_1(i,v)\} & \text{if } u \neq v, \end{cases} \tag{7.6}$$

$$\frac{\partial^2 L(\underset{\sim}{\xi})}{\partial \alpha_u \partial \beta_v} = \begin{cases} \sum_{i=1}^{n} \{\bar{h}_{12}(i,u) - \bar{g}_1(i,u)\bar{g}_2(i,u)\} & \text{if } u = v \\ \sum_{i=1}^{n} \{\bar{d}_{12}(i,u,v) - \bar{g}_1(i,u)\bar{g}_2(i,v)\} & \text{if } u \neq v, \end{cases} \tag{7.7}$$

$$\frac{\partial^2 L(\underset{\sim}{\xi})}{\partial\beta_u \partial\beta_v} = \begin{cases} \sum_{i=1}^{n} \{\bar{h}_{22}(i,u) - \bar{g}_2^2\,(i,u)\} & \text{if } u = v \\ \sum_{i=1}^{n} \{\bar{d}_{22}(i,u,v) - \bar{g}_2(i,u)\bar{g}_2(i,v)\} & \text{if } u \neq v, \end{cases} \tag{7.8}$$

for $u,v = 1,\ldots,k$.

For any pair $(\underset{\sim}{\xi}, \underset{\sim}{\xi}^\circ)$, define the posterior expectations of derivativies and their products,

$$\tilde{g}_s(i,u) = \int g_s(i,u,\theta)\; p(\theta|\underset{\sim}{y}_i,\underset{\sim}{\xi}^\circ)\,d\theta, \tag{7.9}$$

$$\tilde{h}_{st}(i,u) = \int h_{st}(i,u,\theta)\; p(\theta|\underset{\sim}{y}_i,\underset{\sim}{\xi}^\circ)\,d\theta, \tag{7.10}$$

$$\tilde{d}_{st}(i,u) = \int g_s(i,u,\theta)g_t(i,u,\theta)p(\theta|\underset{\sim}{y}_i,\underset{\sim}{\xi}^\circ)\,d\theta\;. \tag{7.11}$$

These integrals are indentical to (7.1) - (7.3) when $\underset{\sim}{\xi} = \underset{\sim}{\xi}^\circ$, but are generally not identical to them when $\underset{\sim}{\xi} \neq \underset{\sim}{\xi}^\circ$. The derivatives of $T(\underset{\sim}{\xi})$ may then be summarized by

$$\frac{\partial T(\underset{\sim}{\xi})}{\partial\alpha_u} = \sum_{i=1}^{n} \tilde{g}_1(i,u), \tag{7.12}$$

$$\frac{\partial T(\underset{\sim}{\xi})}{\partial\beta_u} = \sum_{i=1}^{n} \tilde{g}_2(i,u), \tag{7.13}$$

$$\frac{\partial^2 T(\underset{\sim}{\xi})}{\partial\alpha_u \partial\alpha_v} = \begin{cases} \sum_{i=1}^{n} \{\tilde{h}_{11}(i,u) - \tilde{d}_{11}(i,u)\} & \text{if } u = v \\ 0 & \text{if } u \neq v, \end{cases} \tag{7.14}$$

$$\frac{\partial^2 T(\underset{\sim}{\xi})}{\partial\alpha_u \partial\beta_v} = \begin{cases} \sum_{i=1}^{n} \{\tilde{h}_{12}(i,u) - \tilde{d}_{12}(i,u)\} & \text{if } u = v \\ 0 & \text{if } u \neq v, \end{cases} \tag{7.15}$$

$$\frac{\partial^2 T(\underset{\sim}{\xi})}{\partial\beta_u \partial\beta_v} = \begin{cases} \sum_{i=1}^{n} \{\tilde{h}_{22}(i,u) - \tilde{d}_{22}(i,u)\} & \text{if } u = v \\ 0 & \text{if } u \neq v, \end{cases} \tag{7.16}$$

$u,v = 1,\ldots,k.$

We note that the numerical solution to (3.2) and the evaluation of $I(\hat{\underset{\sim}{\xi}})$ are quite sensitive to the accuracy of the numerical approximation of integrals. A typical integral is a posterior expectation and, except for a constant factor, has the form

$$\int H(\theta)\ p(\theta) \prod_{j=1}^{k} P(y_{ij}|\theta,\alpha_j,\beta_j)\,d\theta, \tag{7.17}$$

where the function H varies from integral to integral. The missing constant factor is the reciprocal of this integral when $H(\theta) \equiv 1$. Since $p(\theta)$ is the standard normal pdf, the integral has the Mth oder Gauss-Hermite approximation

$$\pi^{-1/2} \sum_{\ell=1}^{M} H(\sqrt{2}\, x_\ell) \prod_{j=1}^{k} P(y_{ij} | \sqrt{2}\, x_\ell, \alpha_j, \beta_j) w_\ell \qquad (7.18)$$

where $(x_1, w_1), \ldots, (x_m, w_M)$ are the nodes and weights. (See Strout and Secrest 1966). Since the nodes are based on the prior of θ_i and not its posterior, there is some loss in accuracy when the posterior is displaced from the prior. To avoid this loss one can select the nodes around the posterior mean m_i using the posterior standard deviation c_i as the scale factor. To find m_i and c_i we first use (7.18) with $H(\theta) = \theta$ for m_i and $H(\theta) = (\theta - m_i)^2$ for c_i^2. Then the Gauss-Hermite approximation to (7.17) becomes

$$\sqrt{2}\, c_i \sum_{\ell=1}^{M} w_\ell\, H(m_i + \sqrt{2}\, c_i\, x_\ell) \exp(x_\ell^2)$$

$$\times\, p(m_i + \sqrt{2}\, c_i\, x_\ell) \prod_{j=1}^{k} P(y_{ij} | m_i + \sqrt{2}\, c_i\, x_\ell, \alpha_j, \beta_j).$$

FIGURE 1

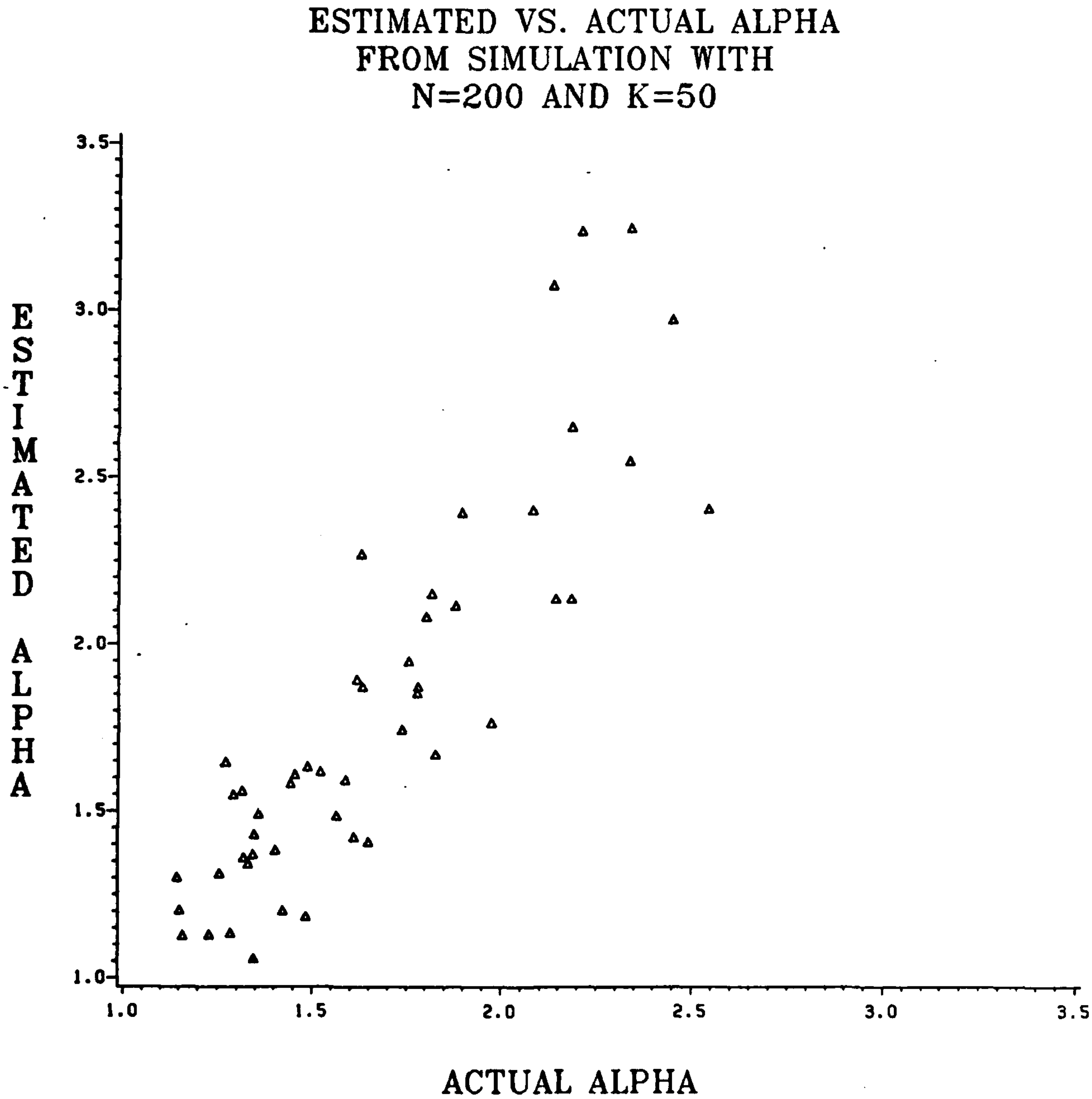
ESTIMATED VS. ACTUAL ALPHA
FROM SIMULATION WITH
N=200 AND K=50
ESTIMATED ALPHA
ACTUAL ALPHA
1.0
1.5
2.0
2.5
3.0
3.5

FIGURE 2

ESTIMATED VS. ACTUAL BETA
FROM SIMULATION WITH
N=200 AND K=50

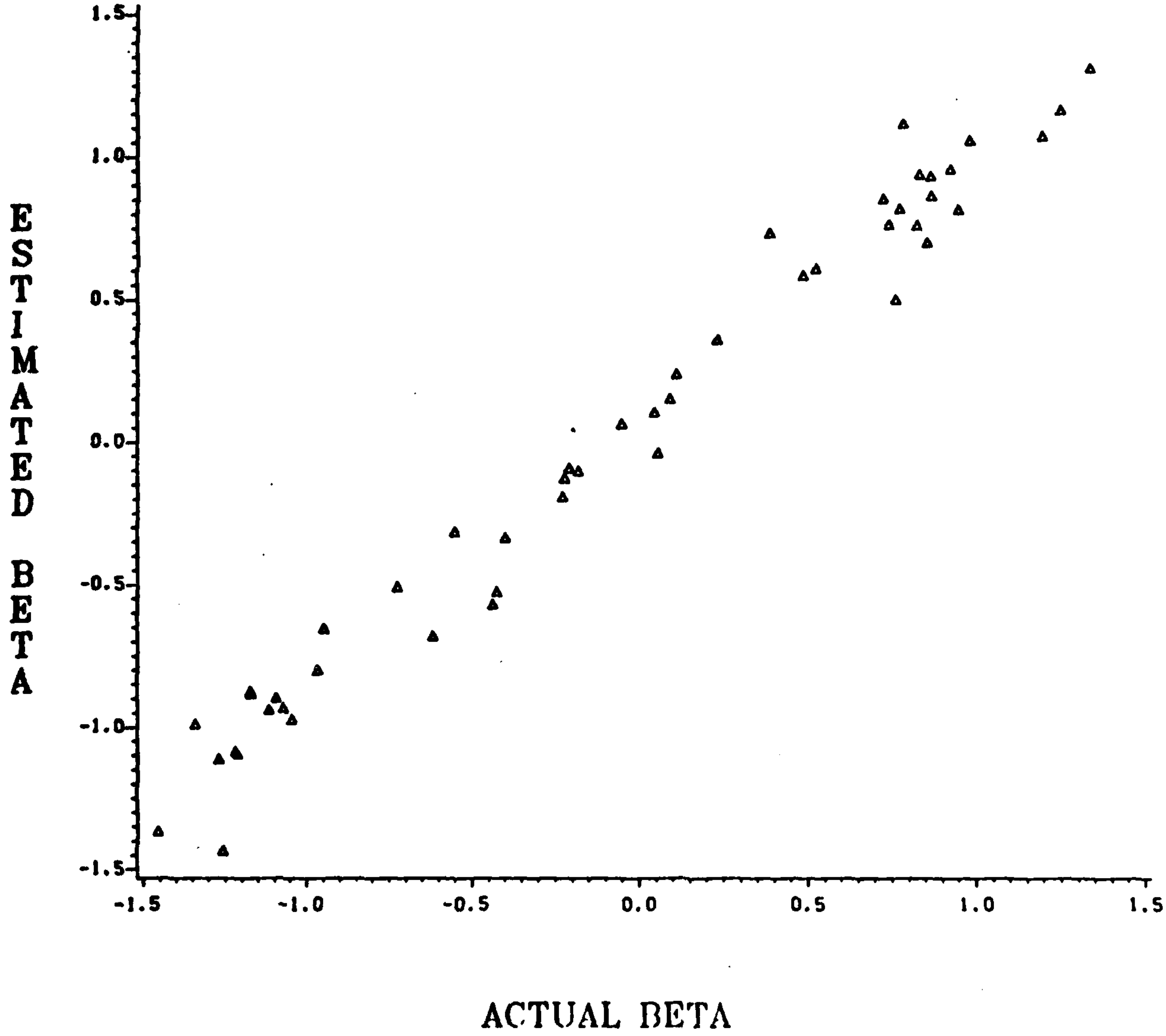

FIGURE 3

ESTIMATED AND ACTUAL THETAS
FROM SIMULATION WITH
N=200 AND K=50

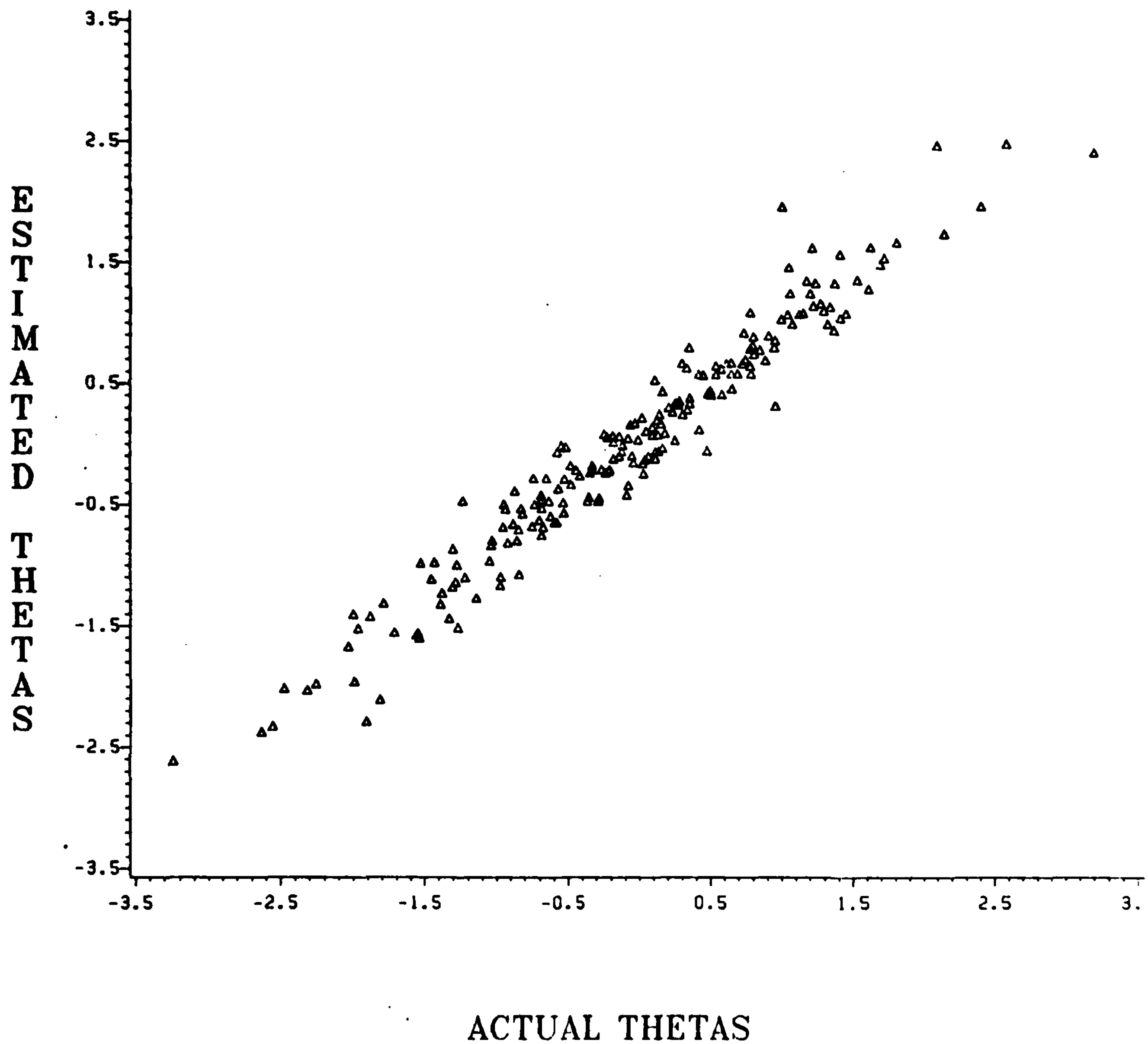

TABLE I

PARAMETER ESTIMATES AND COVARIANCES

FOR ARTHRITIS EXAMPLE

Item	$\hat{\alpha}$	$\hat{\beta}$	$Var(\hat{\alpha})$	$Var(\hat{\alpha},\hat{\beta})$	$Var(\hat{\beta})$
1	1.14	-2.44	0.1287	0.1910	0.3490
2	1.96	-1.69	0.1837	0.0381	0.0310
3	0.62	-3.96	0.0722	0.4022	2.5197
4	1.84	-1.64	0.1552	0.0365	0.0353
5	1.30	-2.03	0.1124	0.0988	0.1431
6	1.41	-1.86	0.0981	0.0508	0.0754
7	1.31	-1.85	0.0983	0.0700	0.1031
8	2.67	-1.28	0.3574	-0.0004	0.0111
9	1.20	-1.85	0.0765	0.0579	0.1033
10	2.12	-1.26	0.1855	0.0123	0.0192
11	1.12	-1.84	0.0715	0.0625	0.1205
12	0.73	-2.40	0.0436	0.1027	0.3838
13	1.05	-1.77	0.0562	0.0457	0.1053
14	0.95	-1.86	0.0538	0.0592	0.1467
15	0.65	-2.43	0.0418	0.1170	0.4909
16	1.65	-1.21	0.0983	0.0064	0.0253
17	0.70	-2.21	0.0491	0.1162	0.4076
18	0.36	-3.89	0.0370	0.3704	4.1652
19	0.69	-1.74	0.0431	0.0802	0.2588
20	0.52	-2.18	0.0400	0.1425	0.6913
21	1.52	-0.89	0.0728	-0.0030	0.0220
22	1.13	-0.98	0.0567	0.0151	0.0436
23	1.11	-0.96	0.0536	0.0150	0.0443
24	0.67	-1.36	0.0432	0.0643	0.1909
25	0.51	-1.45	0.0365	0.0871	0.3493
26	0.68	-0.98	0.0374	0.0363	0.1159
27	0.60	-0.91	0.0416	0.0523	0.1598
28	0.52	-1.05	0.0352	0.0593	0.2250
29	1.10	-0.39	0.0563	0.0058	0.0338
30	1.06	-0.40	0.0575	0.0085	0.0361
31	0.91	-0.33	0.0462	0.0081	0.0444
32	0.26	-1.06	0.0331	0.1350	0.9367
33	0.77	-0.17	0.0480	0.0074	0.0545
34	0.70	-0.19	0.0452	0.0094	0.0648
35	0.61	-0.18	0.0419	0.0121	0.0831
36	0.67	0.04	0.0478	-0.0003	0.0664
37	1.00	0.08	0.0649	-0.0034	0.0360
38	1.29	0.11	0.0954	-0.0035	0.0258
39	0.68	0.44	0.0506	-0.0242	0.0770
40	1.01	0.38	0.0758	-0.0176	0.0400
41	0.46	1.18	0.0466	-0.1052	0.3755
42	0.42	1.33	0.0442	-0.1222	0.4979
43	0.30	2.17	0.0444	-0.2974	2.3145
44	0.75	1.28	0.0712	-0.0952	0.1897
45	0.68	1.71	0.0734	-0.1503	0.3862
46	0.54	2.48	0.0686	-0.2752	1.2395
47	0.44	3.85	0.0840	-0.6664	5.5250

TABLE II

COVARIANCES AND STANDARDIZED ERRORS

FROM SIMULATION

Item	$\widehat{Var}(\hat{\alpha})$	$\widehat{Cov}(\hat{\alpha},\hat{\beta})$	$\widehat{Var}(\hat{\beta})$	Standardized Error $\hat{\alpha}$	Standardized Error $\hat{\beta}$
1	0.0913	0.0303	0.0320	0.33	0.52
2	0.0492	0.0220	0.0388	-0.14	1.78
3	0.1395	0.0200	0.0164	0.75	1.23
4	0.1042	0.0369	0.0347	-0.48	-0.96
5	0.3488	0.0178	0.0088	1.59	1.42
6	0.4080	0.0176	0.0082	1.42	1.29
7	0.1349	0.0100	0.0121	0.91	2.74
8	0.1032	0.0115	0.0151	0.75	2.33
9	0.1747	0.0107	0.0108	0.77	1.72
10	0.0581	0.0153	0.0268	0.24	1.19
11	0.0515	0.0185	0.0333	-1.31	0.77
12	0.0714	0.0167	0.0237	-0.28	0.48
13	0.1907	0.0057	0.0089	0.49	1.79
14	0.0746	0.0055	0.0160	0.57	2.32
15	0.1534	-0.0029	0.0086	-0.35	2.35
16	0.0496	0.0092	0.0255	0.24	-0.41
17	0.3044	-0.0112	0.0064	1.87	2.93
18	0.0561	0.0051	0.0202	0.05	-0.94
19	0.0855	0.0215	0.0166	-0.70	-0.78
20	0.0572	-0.0009	0.0178	-0.07	0.47
21	0.0589	-0.0052	0.0170	-0.77	0.28
22	0.0658	-0.0064	0.0155	1.01	0.75
23	0.0666	-0.0076	0.0155	0.96	0.94
24	0.0519	-0.0057	0.0195	0.71	0.57
25	0.1745	-0.0192	0.0101	1.11	1.12
26	0.0787	-0.0138	0.0150	0.03	0.49
27	0.0587	-0.0085	0.0176	0.34	-0.72
28	0.0905	-0.0161	0.0143	0.93	0.50
29	0.0390	-0.0129	0.0320	-1.39	0.70
30	0.0727	-0.0225	0.0195	0.55	0.92
31	0.0443	-0.0288	0.0440	-1.35	1.65
32	0.0495	-0.0251	0.0330	-0.98	0.55
33	0.1206	-0.0341	0.0197	-0.01	0.60
34	0.1315	-0.0464	0.0258	-0.12	0.81
35	0.0687	-0.0346	0.0316	0.52	0.11
36	0.0568	-0.0216	0.0261	0.19	-1.62
37	0.0781	-0.0400	0.0321	0.03	0.26
38	0.0716	-0.0520	0.0510	-0.89	1.47
39	0.0974	-0.0392	0.0261	0.25	-0.40
40	0.0647	-0.0433	0.0426	0.11	0.50
41	0.1214	-0.0403	0.0222	0.69	-1.04
42	0.0794	-0.0460	0.0370	0.50	0.35
43	0.1112	-0.0482	0.0287	0.58	-0.01
44	0.3021	-0.0824	0.0249	0.95	0.19
45	0.1651	-0.0559	0.0238	1.23	-0.85
46	0.0522	-0.0511	0.0533	-0.42	0.33
47	0.0909	-0.0608	0.0442	1.25	-0.57
48	0.0539	-0.0566	0.0714	-0.63	-0.31
49	0.1972	-0.1049	0.0475	1.45	-0.12
50	0.1587	-0.1260	0.0814	0.24	0.52

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